

# Inertial Mass Lab

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## 1 Data

The following data were observed during this lab:

The width of the flag was 3.10 cm, or 0.0310 m.

The gravitational mass of the cart was 520.30 g, or 0.52030 kg.

Photogate 1 time (s)	Velocity 1 (m/s)	Photogate 2 time (s)	Velocity 2 (m/s)
0.2608	0.1996	1.4393	0.4859
0.9821	0.2006	2.2292	0.4850
0.9261	0.1809	2.1385	0.4410
0.4526	0.1863	1.7412	0.4400
3.3735	0.1920	4.6309	0.4512
0.3381	0.1893	1.5475	0.5152
2.7711	0.1878	4.0000	0.4373
0.2553	0.1894	1.5452	0.4336
0.0829	0.2044	1.2297	0.5041
0.5276	0.2152	1.6280	0.5226
0.3555	0.1813	1.6757	0.4348
0.2512	0.2080	1.4677	0.4506
0.7294	0.2196	1.8186	0.5263
0.4206	0.1843	1.7181	0.4410
0.3110	0.2153	1.4987	0.4594
0.6138	0.1857	1.9030	0.4365
0.6023	0.1437	2.1037	0.4274
0.1467	0.1473	1.6370	0.4274
1.1987	0.1497	2.2589	0.4320
0.1909	0.1421	1.5299	0.4805

## 2 Analysis

This tabulation can be automated by the TI-83+ graphing calculator, using the following steps:

1. Input the data collected into the first four lists.

The image shows a TI-83+ calculator screen with the following data entered:

L3	L4	L5
1.4393	.4859	-----
2.2292	.485	
2.1385	.441	
1.7412	.44	
4.6309	.4512	
1.5475	.5152	
4	.4373	

Below the screen, the text "L5 =" is visible, indicating the next step in the process.

2. Select the fifth list as such:

L3	L4	L5	5
1.4393	.4859		
2.2292	.485		
2.1385	.441		
1.7412	.44		
4.6309	.4512		
1.5475	.5152		
4	.4373		

L5 = (L4 - L2) / (L3 - ...)

3. Input an equation relating list 5 to lists 1 through 4:

The median can be calculated using the calculator's built-in 1-Var Stats function, which can be accessed by pressing STAT-RIGHT-1-2ND-5-ENTER. Shortly, the calculator will display a few meaningful data. In our case, we are looking for the data shown in the following screen shot:

```

1-Var Stats
n=20
minX=.17849599
Q1=.1973611696
Med=.210336405
Q3=.2636902809
maxX=.66161381

```

From here, we can subtract  $Q_1$  from  $Q_3$  (as a sample, .263-.197) to get the IQR, which is about 0.06633. We can also convince the 83+ to create a box plot with the following settings:

```

Plot1 Plot2 Plot3
Off
Type: [L] [U] [D]
Xlist:L5
Freq:1
WINDOW
Xmin=.13018420...
Xmax=.70992558...
Xscl=.1
Ymin=0
Ymax=1
Yscl=1
Xres=1

```

It appears that there are a few outliers that would heavily skew a mean. That would insinuate that a median would be better than a mean.

Using that median, and the formula  $M_{cart} = (0.245/a) - 0.025$ , then the inertial mass of the cart is 1.1398 kg. The sample, of course, is that  $\frac{0.245}{.21033...} - 0.025 = 1.1398$ . Obviously this is inaccurate. I might imagine from reading the rest of this lab that the inaccuracy is due to friction. Since the mass is *larger* than the expected mass, the acceleration is *smaller* than the expected acceleration, so friction seems to be a plausible explanation.

To derive a formula for the frictional force, we need to realize that the frictional acceleration is the expected acceleration minus the true acceleration. In our case, we can find the expected acceleration as such:  $a = 0.245 / (M_{cart} + 0.025)$ . Plugging in our known gravitational mass for  $M_{cart}$ , we get an  $a_{expected}$  of .4492m/s<sup>2</sup>. (The equation looks like this:  $\frac{0.245}{.5203+0.025} = .4492$ .) Our actual acceleration was .2103m/s<sup>2</sup>, so the frictional acceleration was .2389m/s<sup>2</sup>. Multiply that by the mass plus the hanging mass (we'll use the gravitational mass for the moment), and the force is .1303N. So, in reality, the equation for this is:  $f_{friction} = ((M_{hanging} * g) / (M_{grav} + M_{hanging}) - a_{actual}) * (M_{hanging} + M_{grav})$ . On the calculator, that would be input as  $((0.0245) / (.5203 + 0.025) - .2103) * (.5203 + 0.025)$ , and should hopefully give the aforementioned .1303N.

It seems to me that to get the coefficient of friction, we just divide the force of friction pulling back by the normal force. In our case, the normal force is the weight of the cart, expressed as  $M_{cart} * g$ . In our case, we'd be using  $.5203 * 9.8 = 5.098N$ . Now dividing the force of friction by the weight ( $f_{friction} / f_{normal}$ ), we get the coefficient of friction,  $\mu_{kinetic}$ . As a sample,  $.1303N / 5.098N$  gives us a  $\mu_{kinetic} = 0.02555$ . Incorporating the formulae derived previously into this, we get this ugly beast:

$$\mu_{kinetic} = \frac{\left( \frac{M_{hanging} \times g}{M_{grav} + M_{hanging}} - a_{actual} \right) \times (M_{hanging} + M_{grav})}{M_{cart} \times g}$$

### 3 Limitations analysis

So what were the limitations in this lab? Obviously, the string was very real. The string has weight, which would cause acceleration of the cart. The string's (variable) weight was not accounted for in these calculations, and for that reason, probably made them at least somewhat inaccurate. Further experimentation could probably be done by massing the string using a traditional gravitational mass. Another source of error may have been the failure to notice a 500g weight on the gravitational mass. Although the  $\mu_{kinetic}$  does look to be reasonable, it still seems suspicious that the inertial mass appears to be about 500g heavier than the gravitational mass. The LabPro seems to be a possible source of error, as well. Given all the inherent latencies in the USB protocol, it's entirely feasible that the times were off by  $\frac{1}{10}$  second or more. Although the table was initially checked to be level, the underlying tables are not particularly well known for their stability; on the contrary, they tend to rock back and forth if you look at them funny. This could cause an angle, which could cause an unexpected and unaccounted acceleration. We may also be using an incorrect value of  $g$  for our elevation. We also did not mass the weight holder. Over time, it is feasible that it may have lost some mass to other objects. The gate seemed to sit on top of the cart at an angle; the calipers only measured the width of the gate when untilted, not when tilted. There are limitations inherent to the procedure, as well: the velocity measurements are by no means instantaneous velocity. Ideally, a 0-width flag would be used, but that is not possible in our world.

### 4 Sample calculations

Sample calculations have been provided interspersed throughout the text.